Rolle's Theorem :→ Let, $f(x)$ be a poly. with real coefficients. Between two consecutive real roots of the equⁿ $f(x)=0$ there is at least one real root of the equⁿ $f'(x)=0$.

Proof: - Let, α, β be two consecutive real roots of the equⁿ $f(x)=0$ with multiplicity r and s respectively. Then $(x-\alpha)^r(x-\beta)^s$ is a factor of the poly. $f(x)$.

Let, $f(x) = (x-\alpha)^r(x-\beta)^s\phi(x)$. proof not required.

Then $\phi(\alpha) \neq 0$ and $\phi(\beta) \neq 0$, because otherwise the assumed multiplicity of α, β would be contracted. Also, $\phi(\alpha)$ and $\phi(\beta)$ have the same sign, because otherwise $\phi(x)=0$ would have a real root between α and β and consequently $f(x)=0$ would have a real root between α and β which is a contradiction.

$$\text{Now, } f'(x) = r(x-\alpha)^{r-1}(x-\beta)^s\phi(x) + (x-\alpha)^r \cdot s(x-\beta)^{s-1}\phi(x) + (x-\alpha)^r(x-\beta)^s\phi'(x)$$

$$= (x-\alpha)^{r-1}(x-\beta)^{s-1}\psi(x)$$

where, $\psi(x) = r(x-\beta)\phi(x) + s(x-\alpha)\phi(x) + (x-\alpha)(x-\beta)\phi'(x)$

$$\therefore \psi(\alpha) = r(\alpha-\beta)\phi(\alpha)$$

$$\psi(\beta) = s(\beta-\alpha)\phi(\beta)$$

$\because \phi(\alpha)$ and $\phi(\beta)$ are of the same sign, $\psi(\alpha)$ and $\psi(\beta)$ are of different signs and this shows that the equⁿ $\psi(x)=0$ and consequently the equⁿ $f'(x)=0$ has at least one real root which is the required range of values of k.

Descarte's Rule of signs :→

NOTE :→ In a seq. of real nos. $a_0, a_1, \cdots a_n$, none of which is zero, the signs of two consecutive terms may be same or different. when same sign occurs we say that the elements show a continuation of signs. when the signs are different we say that the elements show a variation of signs.

For example, in the seq. 1, 3, -5, -7, 9, -4, 10, 2 there are 3 continuations and 4 variations of signs.

If some of the elements of a seq. be zero, we ignore their presence in the seq. and count the number of continuations and variations of signs.

for example, in the seq. 1, 3, -2, 0, -3, 0, 4, 0, 0, 7 there are 3 continuations and 2 variations of signs.

Statement :→ The number of +ve roots of an equ^n $f(x)=0$ with real coefficients doesnot exceed the number of variations of signs in the seq. of the coefficients of $f(x)$ and if less by an even number.

the no. of +ve roots, then $v = r + 2k$, where k is a nonnegative integer.

NOTE :→ 1) The no. of (-ve) roots of an eqn $f(x) = 0$ with real coefficients does not exceed the no. of variations of signs in the seq. of coefficients of $f(-x)$ and if less, it less by an even number.

2) If $f(x) = 0$ be an eqn of degree n with real coefficients having no zero root and v, v' are respectively the number of variations of signs in the seq. of coefficients of $f(x)$ and $f(-x)$ s.t. $v + v' < n$, then the eqn $f(x) = 0$ has at least $n - (v + v')$ complex roots.

3) If all the roots of the eqn $f(x) = 0$ be nonzero real and v, v' are respectively the no. of variations of signs in the seq. of coefficients of $f(x)$ and $f(-x)$ then the eqn $f(x) = 0$ has v positive roots and v' negative roots.

between α and β.

NOTE :→ 1) Between two consecutive real roots α', β' of the eqn $f'(x)=0$ there is either no real root or at most one real root of the eqn $f(x)=0$ and such a root must be simple.

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Proof :- Because if $\exists$ two real roots of the eqn $f(x)=0$ say, α, β in between α' and β', then by the th. there would exist a real root γ of the eqn $f'(x)=0$ in between α and β and this would contradict that α', β' are consecutive.

Such a root of $f(x)=0$ must be simple because a multiple root of the eqn $f(x)=0$ must also be a root of $f'(x)=0$.

2) If λ' be the least and μ' be the greatest real root of the eqn $f'(x)=0$, each of the intervals $(-\infty, \lambda')$, (μ', ∞) will contain either no real root or at most one real root of the eqn $f(x)=0$.

3) The Interval (α', β') [are root of $f'(x)=0$] will contain no real root of the eqn $f(x)=0$ if $f(\alpha')f(\beta')>0$ and will contain just one real root of the eqn $f(x)=0$ if $f(\alpha')f(\beta')<0$

4) If all the roots of the eqn $f(x)=0$ be real and distinct, then all the roots of $f'(x)=0$ are also so. Any two consecutive roots of $f'(x)=0$ are separated by a root of